6-3 Proving That a Quadrilateral Is a Parallelogram

Common Core State Standards

G-CO.C.11 Prove theorems about parallelograms . . . the diagonals of a parallelogram bisect each other and its converse . . . **Also G-SRT.B.5**

MP 1, MP 3

Objective To determine whether a quadrilateral is a parallelogram

SOLVE IT! **Getting Ready!**

Can you visualize parallelograms composed of triangles in the pattern?

Each section of glass in the exterior of a building in Macau, China, forms an equilateral triangle. Do you think the window washer's feet stay parallel to the ground as he lands at each level of windows? Explain. (Assume that the bases of the lowest triangles are parallel to the ground.)

MATHEMATICAL PRACTICES

In the Solve It, you used angle properties to show that lines are parallel. In this lesson, you will apply the same properties to show that a quadrilateral is a parallelogram.

Essential Understanding You can decide whether a quadrilateral is a parallelogram if its sides, angles, and diagonals have certain properties.

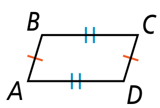
In Lesson 6-2, you learned theorems about the properties of parallelograms. In this lesson, you will learn the converses of those theorems. That is, if a quadrilateral has certain properties, then it must be a parallelogram. Theorem 6-8 is the converse of Theorem 6-3.

take note

Theorem 6-8

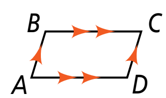
Theorem
If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

If . . .



$\overline{AB} \cong \overline{CD}$
$\overline{BC} \cong \overline{DA}$

Then . . .
$ABCD$ is a ▱



You will prove Theorem 6-8 in Exercise 20.

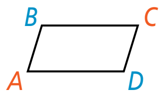
Theorems 6-9 and 6-10 are the converses of Theorems 6-4 and 6-5, respectively. They use angle relationships to conclude that a quadrilateral is a parallelogram.

take note

Theorem 6-9

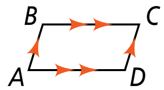
Theorem
If an angle of a quadrilateral is supplementary to both of its consecutive angles, then the quadrilateral is a parallelogram.

If . . .



$m\angle A + m\angle B = 180$
$m\angle A + m\angle D = 180$

Then . . .
$ABCD$ is a ▱

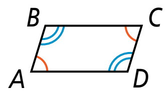


You will prove Theorem 6-9 in Exercise 21.

Theorem 6-10

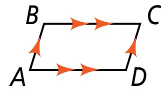
Theorem
If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

If . . .



$\angle A \cong \angle C$
$\angle B \cong \angle D$

Then . . .
$ABCD$ is a ▱



You will prove Theorem 6-10 in Exercise 18.

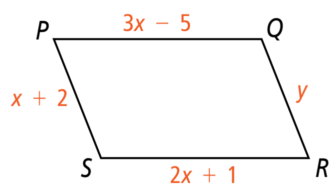
You can use algebra together with Theorems 6-8, 6-9, and 6-10 to find segment lengths and angle measures that assume that a quadrilateral is a parallelogram.

Plan
Which theorem should you use?
The diagram gives you information about sides. Use Theorem 6-8 because it uses sides to conclude that a quadrilateral is a parallelogram.

Problem 1 Finding Values for Parallelograms

GRIDDED RESPONSE

For what value of y must $PQRS$ be a parallelogram?



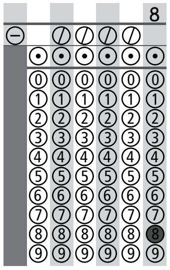
Step 1 Find x.

$3x - 5 = 2x + 1$	If opp. sides are ≅, then the quad. is a ▱.
$x - 5 = 1$	Subtract $2x$ from each side.
$x = 6$	Add 5 to each side.

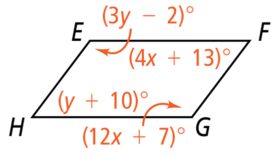
Step 2 Find y.

$y = x + 2$	If opp. sides are ≅, then the quad. is a ▱.
$= 6 + 2$	Substitute 6 for x.
$= 8$	Simplify.

For $PQRS$ to be a parallelogram, the value of y must be 8.



Got It? **1.** Use the diagram at the right. For what values of x and y must $EFGH$ be a parallelogram?



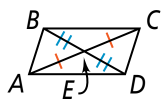
You know that the converses of Theorems 6-3, 6-4, and 6-5 are true. Using what you have learned, you can show that the converse of Theorem 6-6 is also true.

take note

Theorem 6-11

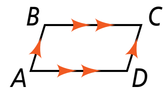
Theorem
If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

If . . .



$\overline{AE} \cong \overline{CE}$
$\overline{BE} \cong \overline{DE}$

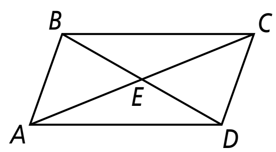
Then . . .
$ABCD$ is a ▱



Proof Proof of Theorem 6-11

Given: $\overline{AC}$ and $\overline{BD}$ bisect each other at E.

Prove: $ABCD$ is a parallelogram.



$\overline{AC}$ and $\overline{BD}$ bisect each other at E.
Given

$\overline{AE} \cong \overline{CE}$, $\overline{BE} \cong \overline{DE}$
Def. of segment bisector

$\angle AEB \cong \angle CED$
Vertical ∠s are ≅.

$\angle BEC \cong \angle DEA$
Vertical ∠s are ≅.

$\triangle AEB \cong \triangle CED$
SAS

$\triangle BEC \cong \triangle DEA$
SAS

$\angle BAE \cong \angle DCE$
Corresp. parts of ≅ △s are ≅.

$\angle ECB \cong \angle EAD$
Corresp. parts of ≅ △s are ≅.

$\overline{AB} \parallel \overline{CD}$
If alternate interior ∠s ≅, then lines are ∥.

$\overline{BC} \parallel \overline{AD}$
If alternate interior ∠s ≅, then lines are ∥.

$ABCD$ is a parallelogram.
Def. of parallelogram

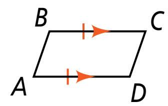
Theorem 6-12 suggests that if you keep two objects of the same length parallel, such as cross-country skis, then the quadrilateral formed by connecting their endpoints is always a parallelogram.

take note

Theorem 6-12

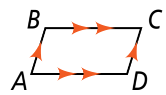
Theorem
If one pair of opposite sides of a quadrilateral is both congruent and parallel, then the quadrilateral is a parallelogram.

If . . .



$\overline{BC} \cong \overline{DA}$
$\overline{BC} \parallel \overline{DA}$

Then . . .
$ABCD$ is a ▱



You will prove Theorem 6-12 in Exercise 19.

Think

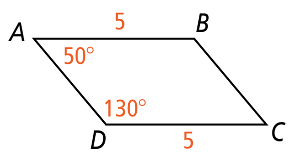
How do you decide if you have enough information?
If you can satisfy every condition of a theorem about parallelograms, then you have enough information.

Problem 2 Deciding Whether a Quadrilateral Is a Parallelogram

Can you prove that the quadrilateral is a parallelogram based on the given information? Explain.

A Given: $AB = 5$, $CD = 5$,
$m\angle A = 50$, $m\angle D = 130$

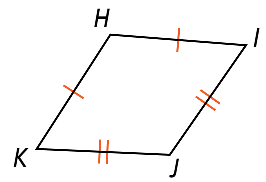
Prove: $ABCD$ is a parallelogram.



Yes. Same-side interior angles A and D are supplementary, so $\overline{AB} \parallel \overline{CD}$. Since $\overline{AB} \cong \overline{CD}$, $ABCD$ is a parallelogram by Theorem 6-12.

B Given: $\overline{HI} \cong \overline{HK}$, $\overline{JI} \cong \overline{JK}$

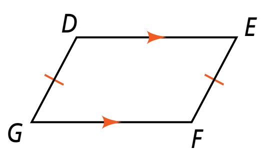
Prove: $HIJK$ is a parallelogram.



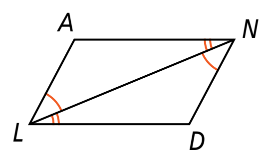
No. By Theorem 6-8, you need to show that both pairs of *opposite* sides are congruent, not consecutive sides.

Got It? 2. Can you prove that the quadrilateral is a parallelogram based on the given information? Explain.

a. Given: $\overline{EF} \cong \overline{GD}$, $\overline{DE} \parallel \overline{FG}$
Prove: $DEFG$ is a parallelogram.



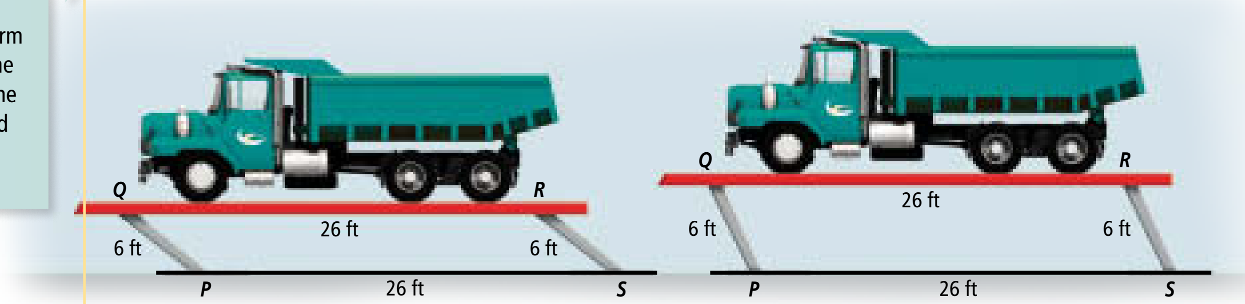
b. Given: $\angle ALN \cong \angle DNL$, $\angle ANL \cong \angle DLN$
Prove: $LAND$ is a parallelogram.



Think
As the arms of the lift move, what changes and what stays the same?
The angles the arms form with the ground and the platform change, but the lengths of the arms and the platform stay the same.

Problem 3 Identifying Parallelograms

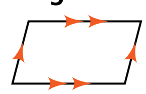
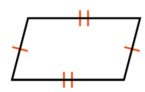
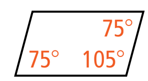
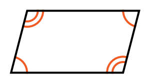
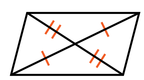
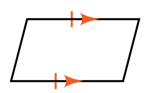
Vehicle Lifts **A truck sits on the platform of a vehicle lift. Two moving arms raise the platform until a mechanic can fit underneath. Why will the truck always remain parallel to the ground as it is lifted? Explain.**



The angles of $PQRS$ change as platform $\overline{QR}$ rises, but its side lengths remain the same. Both pairs of opposite sides are congruent, so $PQRS$ is a parallelogram by Theorem 6-8. By the definition of a parallelogram, $\overline{PS} \parallel \overline{QR}$. Since the base of the lift $\overline{PS}$ lies along the ground, platform $\overline{QR}$, and therefore the truck, will always be parallel to the ground.

Got It? **3. Reasoning** What is the maximum height that the vehicle lift can elevate the truck? Explain.

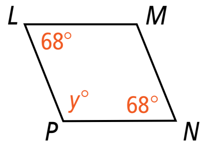
take note — Concept Summary: Proving That a Quadrilateral Is a Parallelogram

Method	Source	Diagram
Prove that both pairs of opposite sides are parallel.	Definition of parallelogram	
Prove that both pairs of opposite sides are congruent.	Theorem 6-8	
Prove that an angle is supplementary to both of its consecutive angles.	Theorem 6-9	
Prove that both pairs of opposite angles are congruent.	Theorem 6-10	
Prove that the diagonals bisect each other.	Theorem 6-11	
Prove that one pair of opposite sides is congruent and parallel.	Theorem 6-12	

Lesson Check

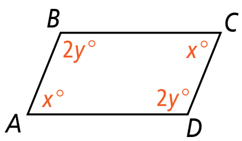
Do you know HOW?

1. For what value of y must $LMNP$ be a parallelogram?

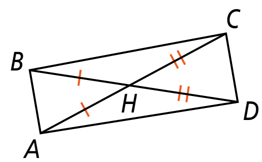


For Exercises 2 and 3, is the given information enough to prove that *ABCD* is a parallelogram? Explain.

2.



3.



Do you UNDERSTAND? MATHEMATICAL PRACTICES

4. **Vocabulary** Explain why you can now write a biconditional statement regarding opposite sides of a parallelogram.

5. **Compare and Contrast** How is Theorem 6-11 in this lesson different from Theorem 6-6 in the previous lesson? In what situations should you use each theorem? Explain.

6. **Error Analysis** Your friend says, "If a quadrilateral has a pair of opposite sides that are congruent and a pair of opposite sides that are parallel, then it is a parallelogram." What is your friend's error? Explain.

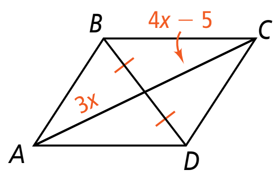
Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

Algebra **For what values of x and y must *ABCD* be a parallelogram?**

See Problems 1 and 2.

7.



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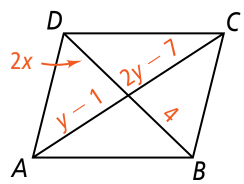

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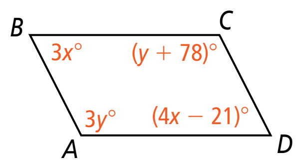

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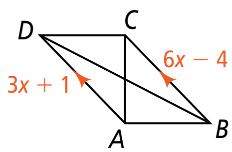

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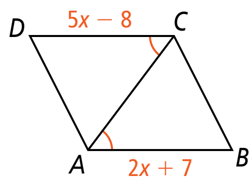

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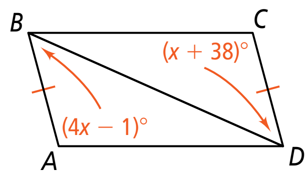
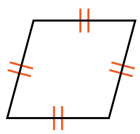


Can you prove that the quadrilateral is a parallelogram based on the given information? Explain.

13.



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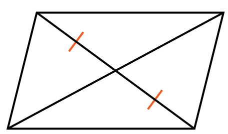

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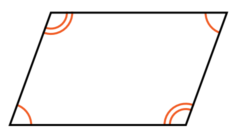
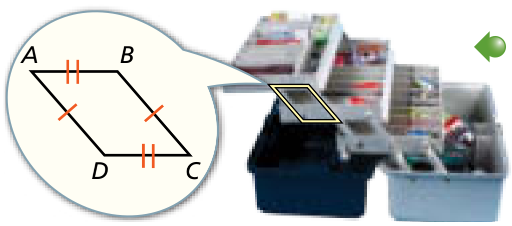


16. **Fishing** Quadrilaterals are formed on the side of this fishing tackle box by the adjustable shelves and connecting pieces. Explain why the shelves are always parallel to each other no matter what their position is.

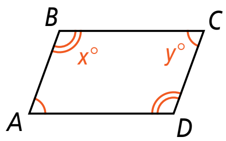
See Problem 3.



B Apply

17. **Writing** Combine each of Theorems 6-3, 6-4, 6-5, and 6-6 with its converse from this lesson into biconditional statements.

18. **Developing Proof** Complete this two-column proof of Theorem 6-10.



Given: $\angle A \cong \angle C$, $\angle B \cong \angle D$

Prove: $ABCD$ is a parallelogram.

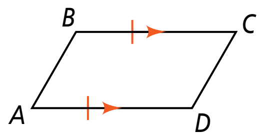
Statements	Reasons
1) $x + y + x + y = 360$	1) The sum of the measures of the angles of a quadrilateral is 360.
2) $2(x + y) = 360$	2) a. ___?___
3) $x + y = 180$	3) b. ___?___
4) $\angle A$ and $\angle B$ are supplementary. $\angle A$ and $\angle D$ are supplementary.	4) Definition of supplementary
5) c. ___?___ $\parallel$ ___?___, ___?___ $\parallel$ ___?___	5) d. ___?___
6) $ABCD$ is a parallelogram.	6) e. ___?___

19. **Think About a Plan** Prove Theorem 6-12.
Proof

Given: $\overline{BC} \parallel \overline{DA}$, $\overline{BC} \cong \overline{DA}$

Prove: $ABCD$ is a parallelogram.

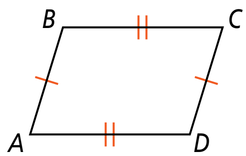
- How can drawing diagonals help you?
- How can you use triangles in this proof?



20. Prove Theorem 6-8.
Proof

Given: $\overline{AB} \cong \overline{CD}$, $\overline{BC} \cong \overline{DA}$

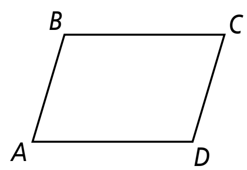
Prove: $ABCD$ is a parallelogram.



21. Prove Theorem 6-9.
Proof

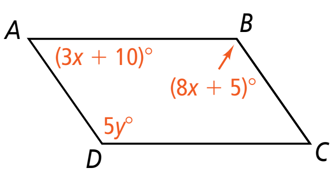
Given: $\angle A$ is supplementary to $\angle B$
$\angle A$ is supplementary to $\angle D$.

Prove: $ABCD$ is a parallelogram.



Algebra **For what values of the variables must *ABCD* be a parallelogram?**

22.



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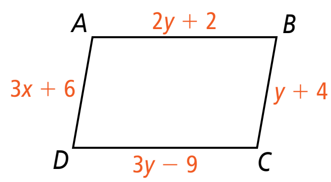

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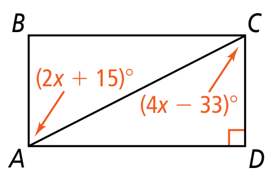
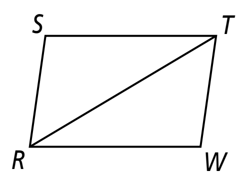


25. **Given:** $\triangle TRS \cong \triangle RTW$
Proof
Prove: $RSTW$ is a parallelogram.

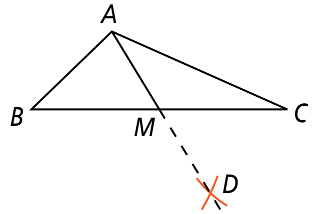


26. **Open-Ended** Sketch two noncongruent parallelograms $ABCD$ and $EFGH$ such that $\overline{AC} \cong \overline{EG}$ and $\overline{BD} \cong \overline{FH}$.

Challenge

27. **Construction** In the figure at the right, point D is constructed by drawing two arcs. One has center C and radius AB. The other has center B and radius AC. Prove that $\overline{AM}$ is a median of $\triangle ABC$.

Proof



28. **Probability** If two opposite angles of a quadrilateral measure 120 and the measures of the other angles are multiples of 10, what is the probability that the quadrilateral is a parallelogram?

Standardized Test Prep

SAT/ACT

29. From which set of information can you conclude that $RSTW$ is a parallelogram?

Ⓐ $\overline{RS} \parallel \overline{WT}$, $\overline{RS} \cong \overline{ST}$

Ⓑ $\overline{RS} \parallel \overline{WT}$, $\overline{ST} \cong \overline{RW}$

Ⓒ $\overline{RS} \cong \overline{ST}$, $\overline{RW} \cong \overline{WT}$

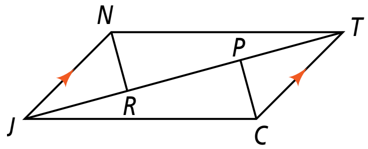
Ⓓ $\overline{RZ} \cong \overline{TZ}$, $\overline{SZ} \cong \overline{WZ}$

Short Response

30. Write a proof using the diagram.

Given: $\triangle NRJ \cong \triangle CPT$, $\overline{JN} \parallel \overline{CT}$

Prove: $JNTC$ is a parallelogram.



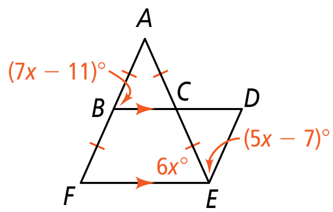
Extended Response

31. Use the figure at the right.

a. Write an equation and solve for x.

b. Is $\overline{AF} \parallel \overline{DE}$? Explain.

c. Is $BDEF$ a parallelogram? Explain.

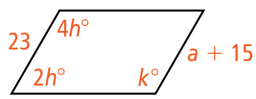


Mixed Review

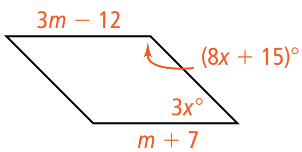
Algebra **Find the value of each variable in each parallelogram.**

See Lesson 6-2.

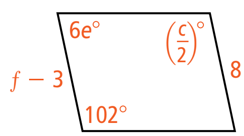
32.



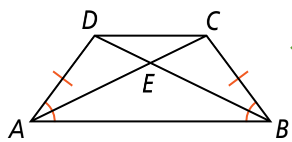
33.



34.



35. Explain how you can use overlapping congruent triangles to prove $\overline{AC} \cong \overline{BD}$.



See Lessons 4-4 and 4-7.

Get Ready! **To prepare for Lesson 6-4, do Exercises 36–44.**

$PACE$ is a parallelogram and $m\angle PAC = 124$. Complete the following.

See Lessons 5-2 and 6-2.

36. $AC = \blacksquare$ **37.** $CE = \blacksquare$ **38.** $PA = \blacksquare$

39. $RE = \blacksquare$ **40.** $CP = \blacksquare$ **41.** $m\angle CEP = \blacksquare$

42. $m\angle EPA = \blacksquare$ **43.** $m\angle ECA = \blacksquare$ **44.** $m\angle ACR = \blacksquare$